

Limits, Derivatives, Integrals

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1 Limits

Evaluate the following limits. You may use l'Hopital's rule!

(a) $\lim_{x \rightarrow \infty} \frac{1}{2x+3}$

(b) $\lim_{x \rightarrow 6} \frac{x-6}{|x-6|}$

(c) $\lim_{x \rightarrow \infty} \frac{x^4-x^2}{2x^3-1}$

(d) $\lim_{x \rightarrow 0^+} \cos(x)^{\frac{1}{x}}$

(e) $\lim_{x \rightarrow \infty} e^{-x} \ln(x)$

(f) $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right)$

(g) $\lim_{x \rightarrow \infty} \sqrt{x^2+1} - x$

(h) $\lim_{x \rightarrow 1} \frac{\sqrt{x+3}-2}{x-1}$

(i) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{1}{2}x^2}{x^3}$

(j) $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+1}}{x}$

2 Derivatives

Find the derivatives of the following functions

- (a) $f(x) = e^{e^{e^x}}$
- (b) $f(x) = \frac{\sin(x)+x}{\ln(x)}$
- (c) $f(x) = x^{\tan(x)}$
- (d) $f(x) = \tan(\sin(\cos(2x)))$
- (e) y' , where $xy + xy^2 + x^2y = 1$
- (f) y' at $(1, 2)$, where $x^2 + 2xy - y^2 + x = 2$
- (g) $f'(x) = \ln(x)^{\ln(x)}$
- (h) $f'''(x)$, where $f(x) = xe^x$

3 Integrals

Evaluate the following integrals.

- (a) $\int_{-3}^3 \sqrt{9 - x^2} dx$
- (b) $\int_0^2 |x - 1| dx$
- (c) $\int \frac{x^4+x^2}{x} dx$
- (d) $\int e^x \sqrt{1 + e^x} dx$
- (e) $\int_0^{\sqrt{\pi}} x \sin(x^2) dx$
- (f) $\int_{-\pi}^{\pi} \frac{x \cos(x)}{1+x^2} dx$
- (g) The average value of $f(x) = \sin(x) \cos(x)^4$ on $[0, \pi]$
- (h) $\int_e^{e^2} \frac{dx}{x \ln(x)}$
- (i) The derivative of $g(x) = \int_{e^x}^{\cos(x)} \ln(1 + x) dx$
- (j) $\int \frac{\tan^{-1}(x)}{1+x^2} dx + \int \frac{1}{\sqrt{1-x^2}} dx$